Telecommun Syst (2010) 44: 171–178
DOI 10.1007/s11235-009-9216-3

Feasibility of optimally assigning channels by exhaustive search in commercial multi-radio wireless mesh networks

Wei Xie · Ying Jun (Angela) Zhang · Mihail L. Sichitiu · Liqun Fu · Yan Yao

Published online: 4 December 2009

Abstract Channel assignment problems in multi-radio wireless mesh networks (WMNs) have been shown to be NP-hard in various scenarios in the literature. By far, most of research efforts have focused on developing efficient *approximation* algorithms that run reasonably fast and provide good quality channel assignment rather than the optimal one. However, from the practical network design and deployment standpoint, engineers care more about whether it is feasible to *optimally* assign channels in the simplest way (i.e., exhaustive search), as most of existing commercial WMNs are of small/medium scale. In this paper, we attempt to answer this practical question. We study the complexity of general channel assignment problems with certain basic and common properties. The results show that the complexity in terms of the number of different possible assignments is exponential in the number of wireless links. Furthermore, we estimate the theoretical runtime of determining the optimal channel assignment by exhaustive search and validate it through experiments. We show that, given certain computing power (e.g., an off-the-shelf notebook PC), it is feasible to optimally solve channel assignment problems in small-scale and medium-scale commercial multi-radio WMNs. In other words, approximation algorithms are not needed for most of current commercial WMNs.

W. Xie (✉) · Y.J. Zhang · L. Fu
Department of Information Engineering, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong
e-mail: wxie@ie.cuhk.edu.hk

M.L. Sichitiu
Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC 27606, USA

Y. Yao
Department of Electronic Engineering, Tsinghua University, Beijing 100084, China

Keywords Wireless mesh networks · Channel assignment · Exhaustive search · Combinatorics

1 Introduction

As a promising wireless broadband technology, wireless mesh network (WMN) is considered one of the best solutions for WLAN *hot zone* coverage [1, 2]. In practice, due to the availability of low-priced radio network interface cards (NICs), more and more commercial mesh routers are equipped with multiple radios working on multiple channels to improve network capacity [3, 4]. However, the number of available wireless channels is quite limited. Therefore, how to assign the limited available channels has become a very important problem in the design and deployment of commercial multi-radio wireless mesh networks.

In recent years, channel assignment problems in wireless mesh networks have been of great interest to both researchers and engineers. A number of channel assignment solutions have been proposed in the literature [5–22], including *dynamic* and *static/quasi-static* approaches [5]. The dynamic approaches [6–8] require channel switching at very fast time scales (e.g., per packet or a handful of packets) and hence require coordination and tight timing synchronization between nodes. In addition, most of the dynamic approaches also require specialized MAC protocols or modification of 802.11 MAC layer. Thus, they are not suitable for use with available commodity hardware. In static/quasi-static approaches [4, 9–13], the algorithms assign channels permanently, or the static assignments can be changed when there are infrequent and significant changes to traffic load or network topology. In quasi-static approaches, the channel-switching delay and traffic measurement overheads are not considered [5].

In addition to the solutions presented above, many *joint* optimization solutions have been proposed recently [13–18]. The related works address various joint optimization problems of channel assignment and other design factors, such as routing, topology control, scheduling, congestion control, and media access control (MAC).

In addition, *partially overlapping* channels have been used together with the non-overlapping channels in some channel assignment approaches to further improve the network capacity of MR-MC WMNs [16, 19–21]. Mohsenian et al. [16] have proposed the use of partially overlapping channels in their Joint Optimal Channel Assignment and Congestion Control (JOCAC) algorithm. The authors have further proposed the channel overlapping matrix and mutual interference matrices to model the non-overlapping and partially overlapping channels [19]. Mishra et al. [20] have formally modeled the degree of overlap between partially overlapping channels. Naveed et al. [21] have used the model in their work to take advantage of partially overlapping channels to further improve spatial reuse and enhance network throughput.

In this paper, we focus on the problems of static/quasi-static assignment of non-overlapping channels to links in the multi-radio WMNs, considering the *generality* and *feasibility* in practical commercial networks. The main purpose of our work is to study the complexity of determining the optimal channel assignment by *exhaustive search* and further to find out whether it is feasible to optimally solve the problems in the simplest way in a given-scale commercial WMN.

In related work, several specific channel assignment problems have been shown to be NP-hard [5, 9, 11, 22]. Raniwala et al. [11] prove the NP-hardness by reducing the *Multiple Subset Sum Problem* [23] to the presented channel assignment problem. Based on the proof in [11], Das et al. [22] show that the directional channel assignment problem is also NP-hard, by breaking the single collision domain used in [11] into smaller collision domains. Marina and Das [9] prove that the channel assignment problem is NP-complete, by showing that the channel assignment problem contains a known NP-complete problem, *minimum edge coloring*, (also called *minimum chromatic index*) [24] as a special case. Subramanian et al. [5] prove the NP-hardness by showing that the channel assignment problem is basically the known NP-hard problem *Max K-cut problem* [25] with the added interface constraint.

It is worth noting that all of the proved NP-hard problems are *specific* channel assignment problems and hence the proof can not be generalized to channel assignment problems with different objectives or constraints. On the other hand, the NP-hardness of channel assignment problems means that it is difficult to determine the optimal channel assignment in large-scale WMNs. For this reason, most works focus on developing efficient *approximation* algorithms that run reasonably fast and provide good quality channel assignment rather than the optimal one. However, in many existing small-scale and medium-scale commercial WMNs, it is possible to identify the optimal channel assignment in a reasonable time. The key question is *for networks of what scale it is feasible to do so?* So far, although a large number of channel assignment solutions have been proposed, few efforts have been made to answer this significant question. This paper is an attempt to find out answers to this question by theoretical analysis and practical experiments. Our contributions are as follows:

- We study the complexity of *pure* channel assignment problems in multi-radio WMNs, rather than the *joint* optimization problems [13–18]. Based on our analytical results, we estimate the theoretical runtime of determining the optimal channel assignment by exhaustive search.
- We focus on the complexity of three *general* and *typical* channel assignment problems with certain basic and common properties. Thus, the results of this work can be applied to similar channel assignment problems.
- We design three exhaustive search-based algorithms to optimally solve the corresponding channel assignment problems and run the software in experiments. By comparing the theoretical runtime with experimental results obtained in practical scenarios, we validate the theoretical estimation of runtime.
- We show that, given certain computing power (e.g., an off-the-shelf notebook PC), it is feasible to determine the optimal channel assignment in practical small-scale and medium-scale commercial multi-radio WMNs.

The rest of this paper is organized as follows. In Sect. 2, the problem formulation is described. In Sect. 3, the complexity analysis of three channel assignment problems is presented in detail. In Sect. 4, the theoretical runtime of determining the optimal channel assignment by exhaustive search is estimated. In Sect. 5, the theoretical result is validated by experiments and the feasibility of optimally assigning channels in commercial WMNs is discussed. Section 6 concludes the paper.

2 Problem formulation

In commercial multi-radio WMNs, it is common to have routers equipped with multiple wireless network interfaces with a distinct channel assigned to each interface. The channel assignment problem, in essence, is to assign available wireless channels to each *wireless network interface*, such that the network capacity is maximized. Both *wireless network interfaces* and *wireless links* can be considered as the basic channel assignment units [11, 16]. In our analysis, we consider the *bi-directional wireless link* as the basic channel assignment unit.

There are two types of traffic in WMNs: network-access traffic between clients and their associated mesh router, and backhaul traffic between mesh routers. In this study, we only discuss the channel assignment problem for backhaul communications and hence only the backhaul connections are considered.

In this work, the network scales of commercial multi-radio WMNs are classified as following three types according to the number of bi-directional wireless links, denoted by L.

- Small-scale multi-radio WMNs: $L \le 20$;
- Medium-scale multi-radio WMNs: $20 < L \le 100$;
- Large-scale multi-radio WMNs: $L > 100$.

Assume that in a backhaul WMN consisting of N wireless mesh routers, there are L bi-directional wireless links and C available non-overlapping channels. Given L and C, the *general channel assignment problem* (G-CAP) is to assign C *different* channels to L *different* links to optimize a certain objective. In practice, link quality of the wireless channel is unknown or uncertain in the early stage of network design and deployment. Therefore, the *practical channel assignment problem* (P-CAP) does not take into account different link qualities of different channels. If the optimization objective of a P-CAP is to maximize network throughput by minimizing aggregate interference between links, then the P-CAP is referred to as an *interference-minimizing* P-CAP (*i-m*P-CAP). In fact, most of the optimization objectives in related works are equivalent to or can be converted equivalently to the one minimizing aggregate interference.

Furthermore, we assume that the computation of the value of the objective function requires polynomial time in L for a given candidate Channel Assignment (CA). Thus the complexity of channel assignment problems is determined by the number of different candidate CAs, denoted by N_{ca}. The main purpose of our analysis presented in Sect. 3 is to determine the different (N_{ca})s in the three aforementioned channel assignment problems.

3 Complexity analysis

3.1 General Channel Assignment Problem (G-CAP)

Given L different links and C different channels, for the $l^{\text{th}}(l = 1, 2, \ldots, L)$ link, it is assigned one of the C channels. Once every link has been assigned a channel, one distinct candidate CA has been constructed.

Since the C channels are different from each other, there are C choices for each link. Hence,

$$(N_{ca})_{G\text{-}CAP} = C \times C \times \cdots \times C = C^L. \tag{1}$$

3.2 Practical Channel Assignment Problem (P-CAP)

As mentioned in Sect. 2, we do not take into account the difference between link qualities of different channels in P-CAP. So we define *equivalent* channels as follows.

Definition 1 Different channels with different frequencies are equivalent if and only if the link qualities of the channels are considered the same.

Since all the available channels are equivalent in P-CAP, different CAs in G-CAP can become equivalent in P-CAP in some cases. An example of two equivalent CAs is shown in Table 1. Given four different links L_1, L_2, L_3, L_4, and three channels CH_1, CH_2, CH_3, two CAs are shown in Table 1. Here, CA_1 and CA_2 are equivalent in P-CAP, while they are considered different in G-CAP.

Given L different links and C *equivalent* channels, the number of different candidate CAs can be determined by using combinatorics.

Let L different links be L elements of a set and let C equivalent channels be C indistinguishable boxes. The P-CAP becomes a set partitioning problem, i.e., how to partition a set of L elements into C indistinguishable boxes in which no empty box is allowed. Therefore, the number of different candidate CAs is equal to the number of partitions.

The number of different candidate CAs in P-CAP is:

$$(N_{ca})_{P\text{-}CAP} = \sum_{k=1}^{C_u} S(L, k), \tag{2}$$

where C_u is the maximum number of the distinct channels that are used in one CA and it is given by

$$C_u = \min(L, C). \tag{3}$$

$S(L, k)$ is the *Stirling number of the second kind* that counts the number of partitions of a set of L elements into k indistinguishable boxes without an empty box [26]. For each integer k with $0 \le k \le L$,

$$S(L, k) = \frac{1}{k!} \sum_{t=0}^{k} (-1)^t \binom{k}{t} (k - t)^L, \tag{4}$$

Table 1 An example of channel assignments

Index	L_1	L_2	L_3	L_4
CA_1	CH_1	CH_2	CH_3	CH_1
CA_2	CH_2	CH_1	CH_3	CH_2

where $\binom{k}{t}$ is the *binomial coefficient* and it is given by

$$\binom{k}{t} = \frac{k!}{t!(k-t)!}. \tag{5}$$

3.3 Interference-minimizing P-CAP (*i-m*P-CAP)

In *i-m*P-CAP, a smaller number of different candidate CAs than $(N_{ca})_{P\text{-}CAP}$ can be obtained. Detailed analysis and deduction are presented as follows.

Lemma 1 *Given L different links and C equivalent channels, for any CA using $k-1$ distinct channels, a CA using k distinct channels that achieves better performance can be found, where $1 < k \leq C_u$.*

Proof Let L_i $(i = 1, 2, \ldots, L)$ be the L different links and CH_i $(i = 1, 2, \ldots, C)$ be the C equivalent but distinct channels. In any CA using $k-1$ distinct channels, denoted by CA_{k-1}, the set consisting of above L links, denoted by $\mathscr{L}$, can be partitioned into $k-1$ non-overlapping subsets as follows.

$$\begin{aligned} &\mathscr{L} = \bigcup_{i=1}^{k-1} \mathscr{L}_i = \mathscr{L}_1 \cup \mathscr{L}_2 \cup \cdots \cup \mathscr{L}_{k-1} \quad \text{and} \\ &\mathscr{L}_i \cap \mathscr{L}_j = \emptyset \quad \text{for } i \neq j \ (i, j = 1, 2, \ldots, k-1), \end{aligned} \tag{6}$$

where the i^{th} link subset $\mathscr{L}_i$ corresponds to a distinct channel CH_i $(i = 1, 2, \ldots, k-1)$. In other words, links in the same subset $\mathscr{L}_i$ are assigned to the same channel CH_i, for any $i = 1, 2, \ldots, k-1$.

According to the definition of C_u given by (3) and the condition provided in Lemma 1, we have

$$1 < k \leq C_u \leq L. \tag{7}$$

Since $k-1$ is smaller than L based on inequality (7), there must be at least one subset that has at least two links according to the *Pigeonhole Principle* [26]. Without losing generality, let $\mathscr{L}_m$ $(1 \leq m \leq k-1)$ be the subset and $\mathscr{L}_m$ can be described as follows.

$$\begin{aligned} &\mathscr{L}_m = \{L_{m_1}, \ldots, L_{m_s}\}, \\ &m_i \in \{1, 2, \ldots, L\} \quad (i = 1, 2, \ldots, s), \quad \text{and} \\ &s = |\mathscr{L}_m| \geq 2, \end{aligned} \tag{8}$$

where s is the size of link subset $\mathscr{L}_m$.

Select randomly one link, say L_{m_q} $(1 \leq q \leq s)$, from $\mathscr{L}_m$ and form another link subset $\mathscr{L}_k$, i.e., $\mathscr{L}_k = \{L_{m_q}\}$. (Note that, the changed $\mathscr{L}_m$ is denoted by $\mathscr{L}'_m$.) By assigning an unused channel CH_k to L_{m_q} of $\mathscr{L}_k$, we construct a new CA using k distinct channels, denoted by CA_k. In CA_k, the link set $\mathscr{L}$ is partitioned into k non-overlapping subsets as follows.

$$\begin{aligned} &\mathscr{L} = \bigcup_{i=1}^{k} \mathscr{L}_i = \mathscr{L}_1 \cup \mathscr{L}_2 \cup \cdots \cup \mathscr{L}_{k-1} \cup \mathscr{L}_k \quad \text{and} \\ &\mathscr{L}_i \cap \mathscr{L}_j = \emptyset \quad \text{for } i \neq j \ (i, j = 1, 2, \ldots, k-1, k), \end{aligned} \tag{9}$$

where $\mathscr{L}_k$ corresponds to the distinct channel CH_k that has not been used by former $k-1$ link subsets.

Thus, the only difference between CA_{k-1} and CA_k is the channel assigned to L_{m_q}, i.e., CH_m in CA_{k-1} and CH_k in CA_k, respectively. By introducing the unused channel CH_k, CA_k reduces the interference between L_{m_q} of $\mathscr{L}_k$ and all the links of $\mathscr{L}'_m$, while they share channel CH_m as elements of $\mathscr{L}_m$ in CA_{k-1}. Meanwhile, the links of rest link subsets, i.e., those except $\mathscr{L}_m$ in CA_{k-1}, remain unchanged performance. Therefore, CA_k can achieve better performance than CA_{k-1}. □

Regarding Lemma 1 and its proof, the following two points are worth noting:

- If different link qualities are taken into consideration for different channels, it is not certain that CA_k can achieve better performance than CA_{k-1}. When a new channel is introduced, although it may reduce the interference, it is possible that the link quality of the new channel is much worse than that of the original one and fails to achieve a better *aggregate* performance in terms of both the subset and the whole network. Therefore, it is necessary for Lemma 1 to have the prerequisite that all channels are equivalent. In other words, Lemma 1 does not hold certainly for G-CAP, while it holds for *i-m*P-CAP.
- If the optimization objective is not equivalent to or can not be converted equivalently to the one minimizing aggregate interference, it is not certain that Lemma 1 holds. In other words, Lemma 1 does not hold certainly for P-CAP, while it holds for *i-m*P-CAP.

Based on Lemma 1, we have the following theorem:

Theorem 1 *Given L different links and C equivalent channels, the optimal CA must use C_u distinct channels.*

Proof The number of different candidate CAs is given by (2). For the k^{th} $(k = 1, 2, \ldots, C_u)$ CA, k different channels are assigned to the L links.

If it is assumed that the optimal CA, denoted by CA_c, has used c distinct channels and c is smaller than C_u, by Lemma 1, the CA using $c+1$ distinct channels achieves better performance. Thus CA_c is not the optimal, which contradicts the assumption that the optimal CA has used $c < C_u$ distinct channels. □

Table 2 Complexity of channel assignment problems

CAPs	Number of Different Candidate CAs
G-CAP	$(N_{ca})_{G\text{-}CAP} = C^L$
P-CAP	$(N_{ca})_{P\text{-}CAP} = \sum_{k=1}^{C_u} S(L,k)$
i-*m*P-CAP	$(N_{ca})_{i\text{-}mP\text{-}CAP} = \begin{cases} 1, & L \le C \\ S(L,C), & L > C \end{cases}$

Table 3 A numerical example of complexity ($C = 3$)

L	4	5	6	7	8	9	⋯
$(N_{ca})_{G\text{-}CAP}$	81	243	729	2187	6561	19683	⋯
$(N_{ca})_{P\text{-}CAP}$	14	41	122	365	1094	3281	⋯
$(N_{ca})_{i\text{-}mP\text{-}CAP}$	6	25	90	301	966	3025	⋯

By Theorem 1 and the definition of C_u in (3), the number of different candidate CAs in the *i*-*m*P-CAP is:

$$(N_{ca})_{i\text{-}mP\text{-}CAP} = S(L, C_u) = \begin{cases} S(L,L) = 1, & L \le C, \\ S(L,C), & L > C. \end{cases} \tag{10}$$

3.4 Complexity comparison

The complexity of channel assignment problems in terms of the number of different candidate CAs is summarized in Table 2. Here, $S(L,k)$ and $S(L,C)$ can be determined by (4). Assuming that three orthogonal channels are available for assignment, a numerical example is given in Table 3.

As shown in Tables 2 and 3, the complexity is exponential in the number of wireless links. Moreover, the N_{ca} in the P-CAP and *i*-*m*P-CAP is much smaller than that in the G-CAP, especially when the number of wireless links becomes larger.

4 Theoretical runtime

In this section, we first deduce the formula for estimating the theoretical runtime of determining the optimal CA by exhaustive search. Then, we present an example in a practical scenario with specific estimation results provided.

4.1 Theoretical estimation

It is necessary to specify the optimization objective of a channel assignment problem before estimating the runtime of determining the optimal CA. According to the definitions of G-CAP and P-CAP, both of them have no limitation on the optimization objective. Hence, we employ the one of *i*-*m*P-CAP for all the three problems, so that we can compare the problems fairly in terms of the runtime. In other words, we assumed that the optimization objective is to minimize aggregate interference between links.

Regarding a given candidate CA, for each of the L links, interference from other $O(L)$ links is calculated to determine the corresponding aggregate interference of the CA. Here, $O(L)$ means that the upper bound on the number of interfering links for each link is equal to L. On the other hand, let K be the time to calculate the interference between two links. Given the computing power, K is a constant and can be determined by tests. So the constant K represents the given computing power in this work. Then, for one candidate CA, the time cost to calculate the objective function is given by $KLO(L)$, which is a second degree polynomial function of L, denoted by $O(L^2)$. Thus, we assume reasonably that the runtime for one candidate CA is KL^2.

Furthermore, we find out the optimal CA by using an *exhaustive search* of all candidate CAs. Hence, according to the (N_{ca})'s determined in Sect. 3 (see Table 2), the theoretical runtime of determining the optimal CA in aforementioned channel assignment problems is:

$$T(K,L,C) = KL^2 \cdot N_{ca}(L,C). \tag{11}$$

4.2 An example

In this example, we consider a practical scenario in which the optimal CA is computed on an off-the-shelf HP notebook PC. The related system configuration information of the notebook is presented as follows:

- Processor: Intel Core Duo T2300E (1.66 GHz, 667 MHz FSB, 2 MB L2 cache);
- Memory: 2 × 512MB DDR2 SDRAM.

As the computing power in this work, K is determined by testing the real time cost to calculate the interference between two links. After running the calculation for 100000 times, we obtain the average value of K as 1.7 µs. Meanwhile, assuming three orthogonal channels are available for assignment, C is equal to 3. This assumption is reasonable because there are only three orthogonal channels in the widely used 802.11b/g bands according to IEEE 802.11 standards. Thus the formula given in (11) can be rewritten as:

$$T(L) = KL^2 \cdot N_{ca}(L). \tag{12}$$

Based on the formula given in (12), we show a graph of the theoretical runtime (i.e., T) versus number of bidirectional wireless links (i.e., L) in Fig. 1.

We observe in Fig. 1 that the runtime of computing the optimal CA by exhaustive search grows exponentially with increasing number of links. On the other hand, for the same increment of number of links, the runtime in G-CAP increases much more than that in P-CAP and *i*-*m*P-CAP.

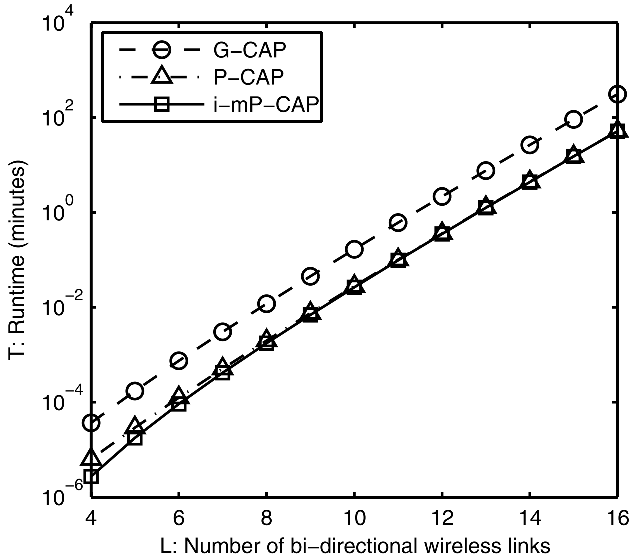

Fig. 1 The theoretical runtime

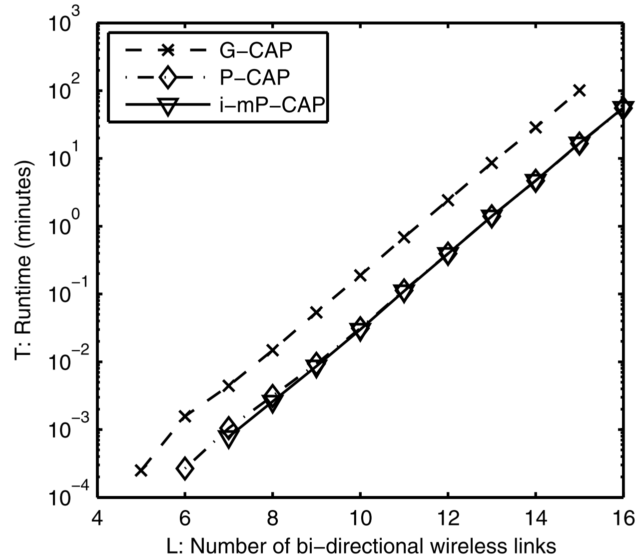

Fig. 2 The experimental runtime

5 Experiment and feasibility discussion

5.1 Experimental validation

Besides the main factors (i.e., K, L, and C) mentioned in Sect. 4, the real runtime cost to determine the optimal CA in practice is also affected by details of specific software design, although the core algorithm is the same (i.e., exhaustive search). Meanwhile, it is impossible for the theoretical estimation to take into account all details of software design. Hence, it is necessary to validate the theoretical results presented in Sect. 4 by experiments in the practical scenario.

Based on the Channel Assignment Tool (CAT) for multi-radio multi-channel WMNs [12], we design three exhaustive search-based algorithms to solve three corresponding channel assignment problems, i.e., G-CAP, P-CAP, and *i*-*m*P-CAP, respectively. The optimal performance of the optimal CA given by CAT is verified in our previous work [12]. In this section, we focus on the real runtime of determining the optimal CA in above three problems by using the CAT with above three algorithms. After being compared with the experimental runtime, the theoretical estimation results can be validated.

We do the experiment in the same practical scenario as that in the example presented in Sect. 4.2. Specifically, we run the software CAT on the same HP notebook PC. Besides, the number of orthogonal channels available for assignment (i.e., C) is also set as 3.

After running the CAT to optimally solve aforementioned channel assignment problems, we record the real experimental runtime. The graph of the experimental runtime versus number of bi-directional wireless links is shown in Fig. 2.

By comparing the theoretical runtime and experimental results shown in Figs. 1 and 2 respectively, we make the following observations.

– The theoretical results and experimental ones are identical in the *relative values* of the runtime in above three channel assignment problems, which can be given by $T_{G\text{-}CAP} > T_{P\text{-}CAP} > T_{i\text{-}mP\text{-}CAP}$.
– Considering each problem separately, the theoretical runtime and experimental results are close to each other. With the growth of network scale, the incremental trend of the runtime in theory is consistent with that in the experiment.

Thus, by experiments in the practical scenario, we validate the theoretical estimation of runtime cost to determine the optimal CA in channel assignment problems.

5.2 Feasibility discussion

According to above analysis and validation, we show that the runtime of optimally solving G-CAP grows much faster than that in P-CAP and *i*-*m*P-CAP when the number of wireless links increases. Regarding all aforementioned channel assignment problems, the runtime is in practice too long, even when the network scale is not very large due to the exhaustive search algorithm. Hence, approximation algorithms are indispensable to channel assignment problems in large-scale wireless mesh networks.

However, we also show that, given certain computing power (e.g., an off-the-shelf notebook PC), it is feasible to determine the optimal CA by exhaustive search for practical commercial small-scale and certain medium-scale mesh networks. Taking the experiment described in this section as an example, the cost runtime in P-CAP and *i-m*P-CAP is in practice acceptable when the number of wireless links is less than twenty. Here, the wireless link is considered as the basic channel assignment unit. But the *real* basic channel assignment unit often includes several wireless links in practical WMNs, due to the *channel dependency* among the links that share a common interface in a mesh router [10, 15]. For example, the *Channel Shared Radio Subset (CSRS)* defined in [12] is a typical practical basic channel assignment unit, which can be determined by the constraints resulting from aforementioned channel dependency. Hence, the number of real assignment unit is in general much less than the number of wireless links. Given the computing power of an ordinary notebook PC with the configuration described in Sect. 4, it is feasible to determine the optimal CA by exhaustive search in commercial medium-scale WMNs as long as the number of the real basic channel assignment unit is less than twenty.

So it is not necessary to design approximation algorithms to provide non-optimal CA for commercial small/medium-scale WMNs. Given certain computing power and network information, we can make a useful judgment in advance by taking the method of *theoretical estimation* that is presented in this paper.

6 Conclusion

In this paper, we study the complexity of three types of channel assignment problems in commercial multi-radio wireless mesh networks. Based on our analytical results, we estimate the theoretical runtime of determining the optimal channel assignment by exhaustive search and validate it through experiments in practical scenarios. We show that, given certain computing power, it is feasible to optimally solve channel assignment problems in practical small-scale and medium-scale commercial multi-radio wireless mesh networks, while approximation algorithms are indispensable in large-scale mesh networks. The results presented in this paper expose fundamental channel assignment problems and provide guidelines for the algorithm design and network planning in commercial wireless mesh networks.

References

1. Gast, M. S. (2005). *802.11 wireless networks: the definitive guide* (2nd ed.). Sebastopol: O'Reilly Media.
2. Akyildiz, I. F., Wang, X., & Wang, W. (2005). Wireless mesh networks: a survey. *Computer Networks Journal (Elsevier)*, *47*(3), 445–487.
3. Bahl, P., Adya, A., Padhye, J., & Wolman, A. (2004). Reconsidering wireless systems with multiple radios. *ACM Computer Communication Review*, *34*(10), 39–46.
4. Kyasanur, P., & Vaidya, N. H. (2005). Capacity of multi-channel wireless networks: Impact of number of channels and interfaces. In *Proc. of ACM MobiCom*. Cologne, Germany, September 2005.
5. Subramanian, A. P., Gupta, H., & Das, S. R. (2007). Minimum interference channel assignment in multi-radio wireless mesh networks. In *Proc. of IEEE SECON*. San Diego, California, June 2007.
6. Bahl, P., Chandra, R., & Dunagan, J. (2004). SSCH: slotted seeded channel hopping for capacity improvement in IEEE 802.11 ad hoc wireless networks. In *Proc. of ACM MobiCom*. Philadelphia, PA, October 2004.
7. So, J., & Vaidya, N. H. (2004). Multi-channel MAC for ad hoc networks: Handling multi-channel hidden terminals using a single transceiver. In *Proc. of ACM MobiHoc*. Tokyo, Japan, May 2004.
8. Kodialam, M., & Nandagopal, T. (2005). Characterizing the capacity region in multi-radio multi-channel wireless mesh networks. In *Proc. of ACM MobiCom*. Cologne, Germany, September 2005.
9. Marina, M., & Das, S. (2005). A topology control approach for utilizing multiple channels in multi-radio wireless mesh networks. In *Proc. of Broadnet*. Boston, MA, October 2005.
10. Raniwala, A., & Chiueh, T. (2005). Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network. In *Proc. of IEEE Infocom*. Miami, FL, March 2005.
11. Raniwala, A., Gopalan, K., & Chiueh, T. (2004). Centralized channel assignment and routing algorithms for multi-channel wireless mesh networks. *ACM Mobile Computing and Communications Review*, *8*(4), 50–65.
12. Xie, W., Xiao, M., Zhao, F., & Yao, Y. (2007). Implementation and evaluation of channel assignment tool for multi-radio multi-channel wireless mesh networks. In *Proc. of IEEE WCNC*. Hong Kong, China, March 2007.
13. Tang, J., Xue, G., & Zhang, W. (2005). Interference-aware topology control and QoS routing in multi-channel wireless mesh networks. In *Proc. of ACM MobiHoc*. Urbana-Champaign, IL, May 2005.
14. Alicherry, M., Bhatia, R., & Li, L. E. (2005). Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks. In *Proc. of ACM MobiCom*. Cologne, Germany, September 2005.
15. Mohsenian-Rad, A. H., & Wong, V. W. S. (2006). Logical topology design and interface assignment for multi-channel wireless mesh networks. In *Proc. of IEEE Globecom*. San Francisco, CA, November 2006.
16. Mohsenian-Rad, A. H., & Wong, V. W. S. (2006). Joint optimal channel assignment and congestion control in multi-radio wireless mesh networks. In *Proc. of IEEE ICC*. Istanbul, Turkey, June 2006.
17. Mohsenian-Rad, A. H., & Wong, V. W. S. (2007). Joint optimal channel allocation, interface assignment, and MAC design for multi-channel wireless mesh networks. In *Proc. of IEEE Infocom*. Anchorage, AK, May 2007.
18. Lin, X., & Rasool, S. (2007). A distributed joint channel-assignment, scheduling and routing algorithm for multi-channel ad hoc wireless networks. In *Proc. of IEEE Infocom*. Anchorage, AK, May 2007.
19. Mohsenian-Rad, A. H., & Wong, V. W. S. (2007). Partially overlapped channel assignment for multi-channel wireless mesh networks. In *Proc. of IEEE ICC*. Glasgow, Scotland, June 2007.
20. Mishra, A., Shrivastava, V., Banerjee, S., & Arbaugh, W. (2006). Partially overlapped channels not considered harmful. In *Proc. of ACM SIGMetric*. Saint Malo, France, June 2006.
21. Naveed, A., Kanhere, S. S., & Jha, S. K. (2007). *Topology control and channel assignment in multi-radio multi-channel wireless mesh networks* (Technical Report UNSW-CSE-TR-0711). The University of New South Wales, Sydney, Australia, May 2007.

22. Das, S. M., Pucha, H., Koutsonikolas, D., Hu, Y. C., & Peroulis, D. (2006). DMesh: Incorporating practical directional antennas in multi-channel wireless mesh networks. *IEEE Journal on Selected Areas in Communications (JSAC)*, *24*(11), 2028–2039, special issue on Multi-Hop Wireless Mesh Networks.
23. Caprara, A., Kellerer, H., & Pferschy, U. (2000). The multiple subset sum problem. *SIAM Journal on Optimization*, *11*.
24. Holyer, I. (1981). The NP-completeness of edge-coloring. *SIAM Journal of Computing*, *10*.
25. Frieze, A., & Jerrum, M. (1997). Improved approximation algorithms for max k-cut and max bisection. *Algoritmica*, *18*.
26. Brualdi, R. A. (2005). *Introductory combinatorics* (4th ed.). London: Pearson Education.

Wei Xie received his B.E. degree in Electronic Engineering from Xiamen University, Xiamen, China, in 2001, and M.S. degree in Electronic Engineering from Chinese Academy of Sciences in 2004, respectively. In 2007, he received his Ph.D. degree in Electronic Engineering from Xiamen University, Xiamen, China. He is currently working as a postdoctoral research assistant in the Department of Information Engineering at the Chinese University of Hong Kong. His research interest is in wireless communications and networking with emphases on network planning and optimization, cross-layer design, and resource allocation.

Ying Jun (Angela) Zhang (S'01–M'05) received B.Eng. degree with Honors in Electronic Engineering from Fudan University, Shanghai China, in 2000, and Ph.D. degree in Electrical and Electronic Engineering from The Hong Kong University of Science and Technology in 2004. Since January 2005, she has been with the Department of Information Engineering, where she is currently an assistant professor.

Dr. Zhang is on the Editorial Boards of IEEE Transactions on Wireless Communications and Willey Security and Communications Journal. She has served as a TPC Co-Chair of Communication Theory Symposium of IEEE ICC 2009, Track Chair of ICCCN 2007, and Publicity Chair of IEEE MASS 2007. She is the IEEE GOLD Technical Conference Program Chair 2008. Her research interests include wireless communications and mobile networks, adaptive resource allocation, cross-layer design and optimization, wireless LAN, and MIMO signal processing.

As the only winner from Engineering Science, Dr. Zhang has won the Hong Kong Young Scientist Award 2006.

Mihail L. Sichitiu was born in Bucharest, Romania. He received a B.E. and a M.S. in Electrical Engineering from the Polytechnic University of Bucharest in 1995 and 1996 respectively. In May 2001, he received a Ph.D. degree in Electrical Engineering from the University of Notre Dame. He is currently employed as an associate professor in the Department of Electrical and Computer Engineering at North Carolina State University. His primary research interest is in Wireless Networking with emphasis on ad hoc networking and wireless local area networks.

Liqun Fu received the B.E. degree (*honors*) in Electronic Engineering from Xiamen University, Xiamen, China, in 2003, and the M.S. degree in Electronic Engineering from Tsinghua University, Beijing, China, in 2006, respectively.

She is currently working toward the Ph.D. degree in the Department of Information Engineering, The Chinese University of Hong Kong. Her current research interest is in wireless communications and networking with emphases on resource allocation, distributed systems and optimization.

Yan Yao graduated from Tsinghua University, Beijing, China in 1962. He is now Professor of Department of Electronic Engineering, Tsinghua University. He has been teaching and researching in the field of wireless & digital communications for more than 40 years. The present research interesting include: broadband transmission, personal communication systems and networks, software radio technology, anti-fading & anti-jamming techniques in wireless communications. He is also Fellow of CIC, senior member of CIE, and senior member of IEEE.