Energy Efficient Transmissions in MIMO Cognitive Radio Networks

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***Abstract*—In this paper, we consider energy efficient transmissions for MIMO cognitive radio networks in which the secondary users coexist with the primary users. We want to optimize the proper time allocations and the beamforming vectors for the secondary users, in order to minimize the total energy consumption of the secondary users while satisfying secondary users' rate requirements and the primary receivers' received interference constraints. The joint time scheduling and beamforming optimization is non-convex and is often highly complex to solve. Fortunately, we show that the optimal time allocation and the optimal beamforming vectors can be found very efficiently in polynomial-time through a proper decomposition. The simulation results show that compared with a simplistic maximum rate transmission policy, our proposed energy-optimal-transmission algorithm can achieve an energy-saving of 26% to 91%, depending on the traffic load of the secondary system.**

***Index Terms*—Cognitive radio networks, MIMO, Energy-efficiency.**

I. Introduction

Cognitive radio, which allows secondary unlicensed users to opportunistically access the spectrum that is under-utilized by the primary licensed users, is a promising approach to improve the spectrum efficiency. In this paper, we focus on energy efficient transmission for cognitive radio networks, in which the secondary users coexist with the primary system with the aid of MIMO techniques.

We consider a cognitive radio network, where the secondary users send traffics to the base station (BS) via Time Division Multiple Access (TDMA). There is no interference among the secondary users. However, the secondary users share the same spectrum with some primary users, and thus the concurrent transmissions of a secondary user and the primary users will cause interference to each other. We aim to choose the proper time allocation and the beamforming vectors for each secondary user, such that the total energy consumption of the secondary users is minimized while satisfying the secondary users' rate requirements and the primary receivers' received interference constraints. Performing energy efficient transmissions in cognitive radio networks is important, as it not only reduces the energy consumption of the secondary users but also alleviates the interference to the primary system.

In the literature, most research on MIMO cognitive radio networks focuses on maximizing the capacity of the secondary system [1]–[3]. On the other hand, the study of energy efficient transmissions for MIMO networks are mostly on traditional MIMO networks where the channel state information (CSI) is available [4]–[6]. The study of energy efficient transmissions for MIMO cognitive radio networks is complicated for two reasons:

1) From the Shannon's capacity formula, increasing the transmission time can reduce the energy consumption for delivering the same amount of traffic. However, in a TDMA network, the total transmission time is shared by multiple secondary users. Increasing the transmission time for one secondary user leads to the reduction of the transmission time for others. Therefore, the energy consumptions of secondary users trade off against each other.
2) For MIMO cognitive radio networks, the primary system may not be aware of the existence of secondary users. Therefore, the secondary system cannot obtain the channel state information (CSI) of the links to primary users. This makes it difficult for the secondary transmitters to perform proper beamforming to avoid interference to primary receivers.

When the CSI is unknown to the secondary system, the deterministic interference constraints cannot be satisfied and thus are not proper. Most previous work deals with the uncertainty using a robust optimization framework, which requires the interference constraints to hold for every possible realization of the channel. Such an approach guarantees the worst case performance and is thus overly conservative. In practice, however, many wireless applications can tolerate occasional outages without affecting users' QoS. This motivates us to consider a more realistic interference requirement, which is to satisfy the interference constraints at the primary receivers with a high probability. Such practical probabilistic constraints are, however, generally hard to deal with mathematically. The corresponding optimization formulation involves joint time scheduling and beamforming, which is non-convex and thus

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difficult to deal with.

In this paper, we show that the energy-optimal time allocation and the optimal beamforming vectors can be computed separately and efficiently. In particular, the optimal beamforming vectors can be obtained very efficiently through a simple matrix eigenvalue-eigenvector computation. Based on the structure of the optimal beamforming vectors, the optimal time allocations can then be found by solving a convex optimization. As a result, the overall problem can be solved very efficiently in polynomial-time.

The remainder of this paper is organized as follows. In Section II, we describe the system model. Section III is the problem formulation. In Section IV, we show that the problem can be decomposed and solved by a polynomial-time algorithm. In Section V, we provide the simulation results. Section VI concludes this paper.

II. System Model

We consider a cognitive radio network with K secondary users and J primary users. The primary links could be always active and thus need to be protected at all times. The network topology for the primary system is general (i.e., can be a cellular network or an ad hoc network). The secondary system is an infrastructure-based network, where the secondary users send uplink traffics via TDMA to the same secondary BS. A sample network topology with two primary links and 20 secondary users is shown in Fig. 1 in Section V.

We use S_k to denote the kth secondary user. Let M_{S_k} denote the number of transmit antennas of the secondary user S_k and N_{BS} denote the number of receive antennas at the secondary BS. Let $\mathbf{H}_{BS,S_k}$ denote the $N_{BS} \times M_{S_k}$ channel matrix from the secondary user S_k to the secondary BS. It is reasonable to assume that $\mathbf{H}_{BS,S_k}$ is known to both S_k and BS. There are J links in the primary network. We use P_j to denote the jth primary link. Let M_{P_j} and N_{P_j} denote the number of transmit antennas and the number of receive antennas of primary user P_j, respectively. Let $\mathbf{H}_{P_i,P_j}$ denote the $N_{P_i} \times M_{P_j}$ channel matrix from the transmitter of the jth primary link to the receiver of the ith primary link. Since the secondary users coexist with the primary users, the secondary user's signal and the primary users' signals may interfere with each other. Let $\mathbf{H}_{P_j,S_k}$ and $\mathbf{H}_{BS,P_j}$ denote the $N_{P_j} \times M_{S_k}$ channel matrix from the secondary user S_k to the receiver of the jth primary link and the $N_{BS} \times M_{P_j}$ channel matrix from the transmitter of the jth primary link to the secondary BS, respectively. We assume Rayleigh fading channels and a rich scattering environment, so that the entries of the channel matrices are independently and identically distributed (i.i.d.) complex Gaussian random variables with a zero mean. The variance of the complex Gaussian variables is half of the path loss from the corresponding transmitter to the corresponding receiver.

Let $\mathbf{u}_{S_k}$ and $\mathbf{v}_{BS_k}$ denote the $M_{S_k} \times 1$ transmit beamforming vector of the secondary user S_k and the $N_{BS} \times 1$ receive beamforming vector of the BS when user S_k is active, respectively. Let $\mathbf{u}_{P_j}$ and $\mathbf{v}_{P_j}$ denote the $M_{P_j} \times 1$ transmit beamforming vector and the $N_{P_j} \times 1$ receive beamforming vector of the primary link P_j, respectively. Without loss of generality, we normalize the receive beamforming vectors such that $\|\mathbf{v}_{BS_k}\|_2^2 = 1$ and $\|\mathbf{v}_{P_j}\|_2^2 = 1$. The cognitive radio network considered here is an interference-limited network, since the secondary users have concurrent transmissions with the primary users. As shown in [7], [8], in an interference-limited network, it is better for each user to transmit one data stream at a time on all its transmit antennas in order to avoid excessive interference to the other links. Thus, let scalars x_{S_k} and x_{P_j} denote the transmit signals of the secondary user S_k and the primary user P_j, respectively. Without loss of generality, we assume $\mathbb{E}[|x_{S_k}|^2] = 1$ and $\mathbb{E}[|x_{P_j}|^2] = 1$. Therefore, the received signal of secondary user S_k after receive beamforming at the secondary BS is

$$y_{BS_k} = \mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k} x_{S_k} + \sum_{j=1}^{J} \mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j} x_{P_j} + \mathbf{v}_{BS_k}^H \mathbf{n}_{BS}, \quad k = 1, \cdots, K,$$

where $\mathbf{n}_{BS}$ is an $N_{BS} \times 1$ circular complex additive Gaussian noise vector with a noise power of N_0 at the secondary BS. The received signal-to-interference-plus-noise ratio (SINR) of the secondary user S_k then is

$$\gamma_{BS_k} = \frac{\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k} x_{S_k}\right|^2}{\sum_{j=1}^{J} \left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j} x_{P_j}\right|^2 + \left|\mathbf{v}_{BS_k}^H \mathbf{n}_{BS}\right|^2} = \frac{\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}\right|^2}{\sum_{j=1}^{J} \left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j}\right|^2 + N_0}, \quad k = 1, \cdots, K. \tag{1}$$

According to the Shannon's capacity formula, the transmission rate of the secondary user S_k is

$$r_{S_k} = w \log(1 + \gamma_{BS_k}) = w \log\left(1 + \frac{\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}\right|^2}{\sum_{j=1}^{J} \left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j}\right|^2 + N_0}\right),$$

where w is the bandwidth used in the secondary system.

The transmit power of the secondary user S_k is

$$p_{S_k} = \|\mathbf{u}_{S_k}\|_2^2, \quad k = 1, \cdots, K,$$

and the secondary user S_k causes an interference to the primary user P_j's receiver at the level of

$$q_{P_j S_k} = \left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k}\right|^2, \quad k = 1, \cdots, K, \quad j = 1, \cdots, J.$$

III. Problem Formulation

Our target is to choose the proper time allocation, and the transmit and receive beamforming vectors for each secondary user, such that the total energy consumption of the secondary system is minimized, while the rate requirement of each secondary user is satisfied and the interference from the secondary system to each receiver in the primary system is below a certain tolerable threshold. Without loss of generality, the TDMA frame length of the secondary system is normalized to be 1. Each secondary user S_k is allocated a time fraction t_{S_k} $(0 \le t_{S_k} \le 1)$ to transmit its data. The transmission rate requirement for each secondary user is R_{S_k}. The transmit power of each secondary user S_k is limited by a maximum transmit power $p_{S_k,\max}$. This problem can be mathematically formulated as follows:

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{K} t_{S_k} \|\mathbf{u}_{S_k}\|_2^2 \\
\text{subject to} \quad & t_{S_k} w \log(1+\gamma_{BS_k}) \ge R_{S_k}, \quad \forall k, & \text{(2a)} \\
& \sum_{k=1}^{K} t_{S_k} \le 1, & \text{(2b)} \\
& \left| \mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k} \right|^2 \le \phi_{P_j}, \quad \forall k, \forall j, & \text{(2c)} \\
& \|\mathbf{u}_{S_k}\|_2^2 \le p_{S_k,\max}, \quad \forall k, & \text{(2d)} \\
\text{variables} \quad & t_{S_k} \ge 0, \quad \forall k, \\
& \mathbf{u}_{S_k}, \quad \forall k, \\
& \mathbf{v}_{BS_k}, \quad \forall k.
\end{aligned}$$

The objective function in (2) is the total energy consumption of all the secondary users. Constraint (2a) guarantees that each secondary user's rate requirement is satisfied. The received SINR γ_{BS_k} in (2a) is a function of variables $\mathbf{u}_{S_k}$ and $\mathbf{v}_{BS_k}$, as shown in (1). Constraint (2b) states that the total time allocated to all the secondary users is no larger than the total TDMA frame length 1. Constraint (2c) ensures that the received secondary interference at each primary receiver is no larger than a tolerable threshold ϕ_{P_j}. Constraint (2d) states that the secondary users have limited transmission power. The variables in (2) are the time fraction variables t_{S_k}, the transmit beamforming vectors $\mathbf{u}_{S_k}$, and the receive beamforming vectors $\mathbf{v}_{BS_k}$ of the secondary users.

We assume that the secondary BS can estimate $\mathbf{H}_{BS,P_j}\mathbf{u}_{P_j}$ for all the primary users P_j by overhearing the transmissions of primary links. Furthermore, as discussed above, the BS also has the knowledge of $\mathbf{H}_{BS,S_k}$ for all the secondary users. However, in a cognitive radio network, the secondary system is usually transparent to the primary system. The primary system would not deliberately provide the channel state information (CSI) to the secondary system. Therefore, the coefficient vectors $\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}$ that appeared in constraint (2c) are not known to the secondary BS.

A. Formulation Simplification

Notice that in formulation (2), the variables $\mathbf{v}_{BS_k}$ only appear in constraint (2a). These variables can be eliminated by exploring the optimal receive beamforming.

As shown in [9], given any transmit beamforming vector $\mathbf{u}_{S_k}$, the optimal receive beamforming vector $\mathbf{v}_{BS_k}^*$ that maximizes the received SINR γ_{BS_k} of user S_k is a minimum-mean-squared-error (MMSE) receiver:

$$\mathbf{v}_{BS_k}^* = \theta_{S_k} \mathbf{B}_{S_k}^{-1} \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}, \quad k = 1, \cdots, K, \tag{3}$$

where $\mathbf{B}_{S_k}$ is an $N_{BS} \times N_{BS}$ matrix given by

$$\mathbf{B}_{S_k} = \sum_{j=1}^{J} \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j} \mathbf{u}_{P_j}^H \mathbf{H}_{BS,P_j}^H + N_0 \mathbf{I}, \quad k = 1, \cdots, K,$$

and θ_{S_k} is the normalized factor given by

$$\theta_{S_k} = \frac{1}{\|\mathbf{B}_{S_k}^{-1} \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}\|_2}, \quad k = 1, \cdots, K,$$

which ensures $\|\mathbf{v}_{BS_k}^*\|_2^2 = 1$.

Substituting (3) into (1), the maximum received SINR γ_{BS_k} with the optimal MMSE receive beamforming is

$$\gamma_{BS_k} = \mathbf{u}_{S_k}^H \mathbf{H}_{BS,S_k}^H \mathbf{B}_{S_k}^{-1} \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}, \quad k = 1, \cdots, K.$$

To simplify notations, let matrix $\mathbf{A}_{S_k}$ denote $\mathbf{H}_{BS,S_k}^H \mathbf{B}_{S_k}^{-1} \mathbf{H}_{BS,S_k}$, which is an $M_{S_k} \times M_{S_k}$ Hermitian matrix. Thus, we have

$$\gamma_{BS_k} = \mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k}, \quad k = 1, \cdots, K. \tag{4}$$

Substituting (4) into constraint (2a), formulation (2) can be simplified to

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{K} t_{S_k} \|\mathbf{u}_{S_k}\|_2^2 \\
\text{subject to} \quad & t_{S_k} w \log\left(1+\mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k}\right) \ge R_{S_k}, \forall k, & \text{(5a)} \\
& \sum_{k=1}^{K} t_{S_k} \le 1, & \text{(5b)} \\
& \left| \mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k} \right|^2 \le \phi_{P_j}, \quad \forall k, \forall j, & \text{(5c)} \\
& \|\mathbf{u}_{S_k}\|_2^2 \le p_{S_k,\max}, \quad \forall k, & \text{(5d)} \\
\text{variables} \quad & t_{S_k} \ge 0, \quad \forall k, \\
& \mathbf{u}_{S_k}, \quad \forall k.
\end{aligned}$$

The receive beamforming vectors $\mathbf{v}_{BS_k}$ are eliminated from the formulation, and the variables in formulation (5) are the time fraction variables t_{S_k} and the transmit beamforming vectors $\mathbf{u}_{S_k}$ of the secondary users.

B. Feasibility

The constraint set in (5) may not always be non-empty. For each secondary user S_k, the maximum feasible instantaneous transmission rate C_{S_k} depends on the maximum transmit power constraint and the interference constraints to the primary receivers. The link capacity C_{S_k} can be computed by solving the following problem

$$\begin{aligned}
\text{maximize} \quad & \gamma_{BS_k} = \mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k} \\
\text{subject to} \quad & \left| \mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k} \right|^2 \le \phi_{P_j}, \quad \forall j, \\
& \|\mathbf{u}_{S_k}\|_2^2 \le p_{S_k,\max}, \\
\text{variable} \quad & \mathbf{u}_{S_k}.
\end{aligned} \tag{6}$$

Let $\gamma_{BS_k,\max}$ denote the optimal objective value of problem (6), then $C_{S_k} = w\log(1+\gamma_{BS_k,\max})$. Formulation (5) is feasible when the traffic load to the secondary system does not exceed its capacity, i.e.,

$$\sum_{k=1}^{K} \frac{R_{S_k}}{C_{S_k}} \leq 1. \tag{7}$$

We assume that there is a call admission control mechanism that guarantees (7), which in turn ensures the feasibility of problem (5). Notice that problem (6) is NP-hard in general [3]. Therefore it is difficult to efficiently compute the link capacity C_{S_k} and check the feasibility of problem (5). In the subsequent section, we will discuss the feasibility condition and the solution method of solving (5) in the case that the secondary BS has no knowledge of $\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}$.

IV. Formulation Decomposition and Optimal Solution

In this section, we will show that there is a closed-form feasibility condition when the secondary BS has no knowledge of $\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}$. Furthermore, the optimal time fractions and the optimal transmit beamforming vectors of problem (5) can be obtained efficiently through a proper decomposition. The key reason for achieving this is that we replace the deterministic constraint (5c) by a probabilistic one that is more meaningful in this scenario.

When the secondary system has no knowledge of vectors $\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}$, these vectors appear random to the secondary users. Thus, the left-hand-side of constraint (5c) (the interference constraint) is random for any given $\mathbf{u}_{S_k}$. The requirement of satisfying the deterministic interference constraint (5c) would easily lead to suboptimal or infeasible solutions. Interestingly, many wireless applications (such as video streaming, voice over IP) can tolerate occasional outages without affecting users' QoS. Thus, in this paper we consider a more realistic requirement, which is to satisfy the interference temperature constraints with a high probability. In other words, the cognitive radio network allows the interference from the secondary transmitters to the primary receivers to exceed the power threshold ϕ_{P_j} with a small outage probability δ_{P_j}. Constraint (5c) is then replaced by

$$\Pr_{\mathbf{H}_{P_j,S_k},\mathbf{v}_{P_j}} \left\{ \left| \mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k} \right|^2 \leq \phi_{P_j} \right\} \geq 1-\delta_{P_j}, \quad \forall k, \forall j, \tag{8}$$

where the probability is taken over both $\mathbf{H}_{P_j,S_k}$ and $\mathbf{v}_{P_j}$.

Since we consider Rayleigh fading channels, the entries of the channel matrix $\mathbf{H}_{P_j,S_k}$ are i.i.d. complex Gaussian random variables with zero mean and a variance of $\frac{\beta_{P_j,S_k}}{2}$, where β_{P_j,S_k} denotes the path loss from the secondary user S_k to the receiver of the jth primary link. Furthermore, because $\mathbf{H}_{P_j,S_k}$ and $\mathbf{v}_{P_j}$ are independent of each other, $\left| \mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k} \right|^2$ follows an exponential distribution with parameter $\frac{1}{\beta_{P_j,S_k} \|\mathbf{u}_{S_k}\|_2^2}$, as shown in [3]. So we have

$$\Pr_{\mathbf{H}_{P_j,S_k},\mathbf{v}_{P_j}} \left\{ \left| \mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k} \right|^2 \leq \phi_{P_j} \right\} = 1-\exp\left(-\frac{\phi_{P_j}}{\beta_{P_j,S_k} \|\mathbf{u}_{S_k}\|_2^2} \right).$$

Therefore, the outage probability constraint (8) is equivalent to

$$\|\mathbf{u}_{S_k}\|_2^2 \leq \frac{-\phi_{P_j}}{\beta_{P_j,S_k} \log \delta_{P_j}}, \quad \forall k, \forall j. \tag{9}$$

Furthermore, after converting the outage probability constraint to (9), we find that (9) can be combined with the maximum transmission power constraint (5d). Let $\lambda_{S_k} = \min\left\{ \frac{-\phi_{P_1}}{\beta_{P_1,S_k} \log \delta_{P_1}}, \cdots, \frac{-\phi_{P_J}}{\beta_{P_J,S_k} \log \delta_{P_J}}, p_{S_k,\max} \right\}$. Constraint (9) and constraint (5d) are equivalent to the following constraint

$$\|\mathbf{u}_{S_k}\|_2^2 \leq \lambda_{S_k}, \quad \forall k = 1, \cdots, K.$$

Therefore, problem (5) can be recast as follows:

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{K} t_{S_k} \|\mathbf{u}_{S_k}\|_2^2 \\ \text{subject to} \quad & t_{S_k} w \log\left(1+\mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k}\right) \geq R_{S_k}, \forall k, \\ & \sum_{k=1}^{K} t_{S_k} \leq 1, \\ & \|\mathbf{u}_{S_k}\|_2^2 \leq \lambda_{S_k}, \quad \forall k, \\ \text{variables} \quad & t_{S_k} \geq 0, \quad \forall k, \\ & \mathbf{u}_{S_k}, \quad \forall k. \end{aligned} \tag{10}$$

A. Formulation Decomposition

In this subsection, we will show that the time fractions t_{S_k} and the transmit beamforming vectors $\mathbf{u}_{S_k}$ can be separately optimized without affecting the overall optimality.

Given any time fraction allocation $(t_{S_1}, \cdots, t_{S_K})$, formulation (10) then reduces to K separate optimization problems among the secondary users. For each secondary user S_k, the optimization problem is given by

$$\begin{aligned} \text{minimize} \quad & \|\mathbf{u}_{S_k}\|_2^2 \\ \text{subject to} \quad & w \log\left(1+\mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k}\right) \geq \frac{R_{S_k}}{t_{S_k}}, && \text{(11a)} \\ & \|\mathbf{u}_{S_k}\|_2^2 \leq \lambda_{S_k}, && \text{(11b)} \\ \text{variable} \quad & \mathbf{u}_{S_k}. \end{aligned}$$

Now, we proceed to show that optimization problem (11) has a closed-form solution by a simple eigenvalue-eigenvector computation. Let $\rho_{S_k,\max}$ denote the largest eigenvalue of $\mathbf{A}_{S_k}$ and $\mathbf{z}_{S_k,\max}$ ($\|\mathbf{z}_{S_k,\max}\|_2^2 = 1$) denote the normalized eigenvector of $\mathbf{A}_{S_k}$ associated with $\rho_{S_k,\max}$. The closed-form solution to (11) is given in the following theorem.

Theorem 1: The necessary and sufficient condition for optimization problem (11) to be feasible is

$$t_{S_k} \geq \frac{R_{S_k}}{w \log\left(\lambda_{S_k} \rho_{S_k,\max} + 1\right)}. \tag{12}$$

When condition (12) is satisfied, the optimal solution to (11) is

$$\mathbf{u}_{S_k}^* = \sqrt{\frac{\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right) - 1}{\rho_{S_k,\max}}}\mathbf{z}_{S_k,\max}. \quad (13)$$

Proof: Matrix $\mathbf{A}_{S_k}$ is a Hermitian matrix. Therefore, matrix $\mathbf{A}_{S_k}$ can be unitarily diagonalized as $\mathbf{A}_{S_k} = \mathbf{Q}_{S_k}\mathbf{\Lambda}_{S_k}\mathbf{Q}_{S_k}^H$, where $\mathbf{Q}_{S_k}$ is a unitary matrix and $\mathbf{\Lambda}_{S_k}$ is a diagonal matrix containing all the eigenvalues of matrix $\mathbf{A}_{S_k}$. So we have

$$\begin{aligned}\mathbf{u}_{S_k}^H\mathbf{A}_{S_k}\mathbf{u}_{S_k} &= \mathbf{u}_{S_k}^H\mathbf{Q}_{S_k}\mathbf{\Lambda}_{S_k}\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k} \\ &= \left(\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k}\right)^H\mathbf{\Lambda}_{S_k}\left(\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k}\right) \\ &\le \rho_{S_k,\max}\|\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k}\|_2^2.\end{aligned}$$

Since matrix $\mathbf{Q}_{S_k}$ is unitary, we further know that $\|\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k}\|_2^2 = \|\mathbf{u}_{S_k}\|_2^2$. So we have

$$\mathbf{u}_{S_k}^H\mathbf{A}_{S_k}\mathbf{u}_{S_k} \le \rho_{S_k,\max}\|\mathbf{u}_{S_k}\|_2^2, \quad (14)$$

where the equality is achieved when $\mathbf{u}_{S_k}$ is an eigenvector of $\mathbf{A}_{S_k}$ corresponding to the maximum eigenvalue $\rho_{S_k,\max}$.

On the other hand, constraint (11a) is equivalent to

$$\mathbf{u}_{S_k}^H\mathbf{A}_{S_k}\mathbf{u}_{S_k} \ge \exp\left(\frac{R_{S_k}}{wt_{S_k}}\right) - 1. \quad (15)$$

According to (14) and (15), we know that if we only consider constraint (11a) in optimizing formulation (11), the minimum value of the objective function in (11) is $\|\mathbf{u}_{S_k}^*\|_2^2 = \frac{\left(\frac{R_{S_k}}{wt_{S_k}}\right)-1}{\rho_{S_k,\max}}$, and the optimal solution is $\mathbf{u}_{S_k}^* = \sqrt{\frac{\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right)-1}{\rho_{S_k,\max}}}\mathbf{z}_{S_k,\max}$. Since constraint (11b) only states that $\|\mathbf{u}_{S_k}\|_2^2$ should be no greater than λ_{S_k}, therefore, (11) is feasible if and only if the minimum value of $\|\mathbf{u}_{S_k}\|_2^2$ satisfies constraint (11b). That is,

$$\frac{\left(\frac{R_{S_k}}{wt_{S_k}}\right) - 1}{\rho_{S_k,\max}} \le \lambda_{S_k} \Rightarrow t_{S_k} \ge \frac{R_{S_k}}{w\log\left(\lambda_{S_k}\rho_{S_k,\max} + 1\right)}.$$

■

According to Theorem 1, the optimal transmit beamforming vectors are functions of the time fraction allocation $(t_{S_1}, \cdots, t_{S_K})$. This enables us to solve optimization problem (10) through a proper decomposition.

Theorem 2: The necessary and sufficient for optimization problem (10) to be feasible is

$$\sum_{k=1}^{K}\frac{R_{S_k}}{w\log\left(\lambda_{S_k}\rho_{S_k,\max} + 1\right)} \le 1. \quad (16)$$

Furthermore, problem (10) can be solved exactly in polynomial time. In particular, the optimal time fractions are the optimal solutions to the convex optimization (17). The optimal transmit beamforming vectors are then given by the closed-form solution in (13).

Proof: Substituting (13) into formulation (10), then (10) becomes

$$\begin{aligned}\text{minimize} \quad & \sum_{k=1}^{K} t_{S_k}\frac{\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right) - 1}{\rho_{S_k,\max}} \\ \text{subject to} \quad & \sum_{k=1}^{K} t_{S_k} \le 1, \\ & t_{S_k} \ge \frac{R_{S_k}}{w\log\left(\lambda_{S_k}\rho_{S_k,\max} + 1\right)}, \quad \forall k, \\ \text{variables} \quad & t_{S_k} \ge 0, \quad \forall k.\end{aligned} \quad (17)$$

The condition for the constraint set of (17) to be non-empty is $\sum_{k=1}^{K}\frac{R_{S_k}}{w\log\left(\lambda_{S_k}\rho_{S_k,\max}+1\right)} \le 1$.

Problem (17) is an optimization problem with the time fraction variables t_{S_k} only. The second order derivative of the objective function in (17) with respect to variable t_{S_k} is

$$\frac{R_{S_k}^2}{w^2\rho_{S_k,\max}t_{S_k}^3}\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right),$$

which is always positive for any nonnegative t_{S_k}. Thus, the objective function in (17) is a convex function. Furthermore, the constraints in (17) are linear. Therefore, problem (17) is a convex optimization problem, which can be solved by the standard interior-point method [10] in polynomial time. ■

B. Optimal Solution

According to Theorem 2, finding the optimal solutions to the optimization problem (10) is straightforward. The secondary BS first checks whether condition (16) is satisfied. If yes, the BS solves the convex optimization problem (17) to obtain the optimal time fractions. Given the optimal time fractions, the optimal transmit beamforming vectors are then computed by (13). After obtaining the optimal transmit beamforming vectors, the optimal receive beamforming vectors can be computed by (3). If condition (16) cannot be satisfied, then the rate requirements of all users in the secondary system cannot be satisfied with the maximum transmit power constraint and the probabilistic interference constraint. In this case, the secondary system needs to perform call admission control to block the newly coming secondary user.

V. Simulation Results

In this section, we carry out simulations to evaluate the performance of the proposed algorithm. We simulate a cognitive radio network with two primary links. The network topology is shown in Fig. 1.

The length of each primary link is 50 meters. The base station of the secondary system is placed at the center of the square area of $300m \times 300m$. The secondary users are uniformly distributed in the square. We adopt the Rayleigh fading channel model and set the path loss exponent to 4. All the transmitters and receivers are equipped with 4 antennas. The bandwidth is 1 MHz. The frame length of the secondary system is normalized to be 1 second. The session

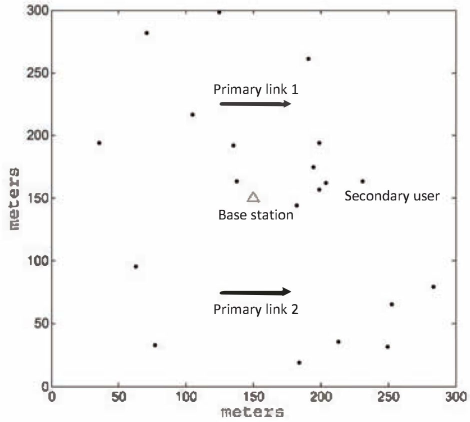

Fig. 1. Network topology with 20 secondary users.

rate requirement of each secondary user is 200 kbps. The maximum output power is 27.5 dBm. The primary users both transmit at the maximum power. The noise power density is -174 dBm/Hz. The tolerable interference power at each primary receiver ϕ_{P_j} is chosen such that $\frac{\phi_{P_j}}{N_0}$ is 20 dB. The outage probability δ_{P_j} is set to be 1%. Each point in the curves is an average of 1000 simulation runs with independent secondary user locations.

We compare the energy consumption performances of the following two transmission policies:

1) Maximum rate transmission: each secondary user transmits at its maximum rate while satisfying the maximum transmit power constraint and the interference temperature constraint to the primary receivers.
2) Energy optimal transmission: the energy-optimal time scheduling and beamforming proposed in this paper.

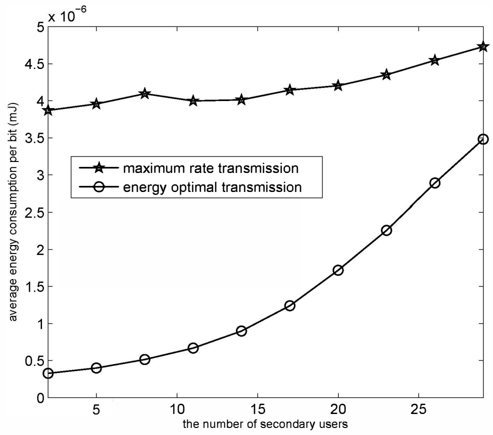

Fig. 2. Average energy consumption per bit of the secondary system vs. the number of secondary users.

Figure 2 shows the energy consumption per bit of the secondary system as a function of the number of secondary users. As we can see, the energy optimal transmission significantly saves the energy consumption. In the maximum rate transmission policy, the average energy consumption per bit does not vary much with the secondary system's traffic load. In the energy-optimal transmission policy, the base station reduces the energy consumption by fully utilizing the time resource. Therefore, the energy consumption per bit is adaptive to the system traffic load. When the traffic load is low (each secondary user has more available time resource), the energy saving becomes more significant. Compared with the maximum rate transmission policy, our proposed algorithm achieves an energy reduction that ranges from 26% (the case of 29 secondary users) to 91% (the case of 2 secondary users).

VI. Conclusion

In this paper, we considered the energy-optimal time allocation and beamforming in MIMO cognitive radio networks, when the secondary system has no channel knowledge on the links to the receivers of the primary system. We showed that the energy optimal beamforming vectors can be found by a simple matrix eigenvalue-eigenvector computation. Based on the closed-form structure of the optimal beamforming solutions, we further showed that the optimal time allocation can be found by solving a convex optimization. Therefore, the overall problem can be solved in polynomial-time with a proper decomposition. The energy saving benefits become more significant when the traffic load of the secondary system is low. The simulation results showed that compared with the simplistic maximum rate transmission policy, our proposed energy-optimal transmission algorithm can achieve an energy-saving of 26% to 91% depending on the traffic load of the secondary system.

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